



**MATHEMATICS SPECIALIST 3CD  
COMMON TEST 5 – Term 3 2010**

**Topic(s):**    *Rectilinear Motion*  
                  *Simple Harmonic Motion*  
                  *Mathematical Reasoning*  
                  *Marginal Analysis*  
                  *Exponential Growth/Decay*

Name: \_\_\_\_\_

Marks: \_\_\_\_\_ / 50

**Instructions:**

- Answer all the questions in the spaces provided
- Casio Classpad Calculator may be used
- External notes are not allowed
- Duration of test: 50 minutes
- This test contributes to 5% of the year (school) mark

1. [8 marks]

Prove by mathematical induction that

$$1 + 4 + 7 + 10 + \dots + (3n-2) = \frac{n(3n-1)}{2}$$

2. [8 marks]

A company manufactures action figures. Its marketing department determines the price-demand function and the cost function defined below:

$$p(x) = 119 - 6x, \text{ for } 1 \leq x \leq 15$$

$$C(x) = 234 + 23x$$

Where  $p$  is the wholesale price per action figure at which  $x$  million action figures can be sold, and  $C$  is in millions of dollars.

- (a) Determine the revenue and profit functions,  $R(x)$  and  $P(x)$ .

2m

[2]

- (b) Verify that  $C'(8) = R'(8)$ , and state the significance of this result.

2m

[2]

- (c) Explain why we can approximate the cost of producing the  $(x + 1)^{\text{th}}$  item using the marginal cost function  $C'(x) \approx C(x+1) - C(x)$ .

2m

[2]

- (d) Find the maximum profit from the sale of the action figures.

2m

[2]

3. [12 marks]

According to Newton's law of Cooling, the temperature  $T$ °C (Celsius) of a hot metal slab left to cool down satisfies the equation

$$\frac{dT}{dt} = -k(T - 20)$$

where  $k$  is a positive constant and  $t$  is measured in minutes.

- (a) In the expression provided above, what does the number 20 represent?

1m

[1]

- (b) After 20 minutes the temperature of the slab is 50° C and after 40 minutes it is 30° C. Use the method of separation of variables to determine  $T$  as a function of  $t$ .

8m

[8]

(c) What is the initial temperature of the slab?

*1m*

[1]

(d) How long will it take for the temperature of the slab to drop to within  $5^{\circ}\text{C}$  of its final temperature?

*2m*

[2]

4. [10 marks]

A particle moves in simple harmonic motion in a straight line with a period of 5 seconds and amplitude of 4 metres. Initially the particle is 1 metre from its equilibrium point and is moving towards it. Determine:

- (a) its distance from its equilibrium point after 3 seconds, to the nearest centimetre.

5m

(b) the total distance travelled in the first 4 seconds, to the nearest centimetre.

5m

[5]

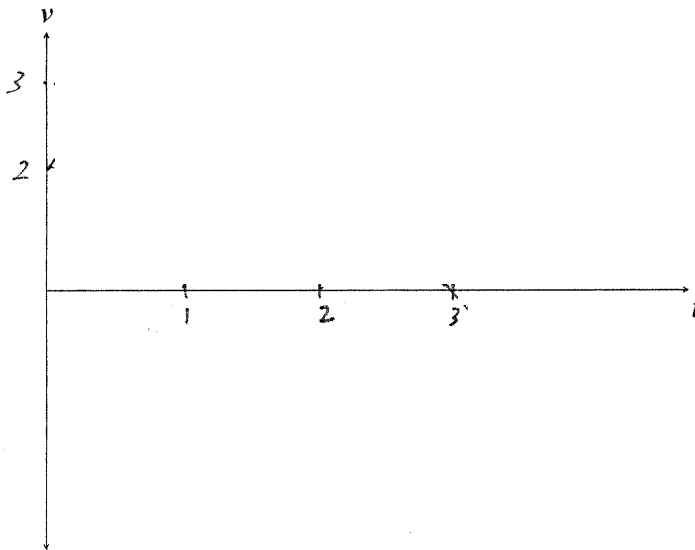
5. [12 marks]

A particle is first observed at time  $t = 0$  and its position at this point is taken as its initial position. The particle moves in a straight line so that its velocity,  $v$ , at time  $t$  is given by:

$$v = \begin{cases} 3 - (t - 1)^2 & \text{for } 0 \leq t \leq 2 \\ 6 - 2t & \text{for } t > 2 \end{cases}$$

(a) On the axes below, sketch the velocity-time graph for  $t \geq 0$ .

3m



answer hidden

[3]

(b) Determine the distance travelled by the particle from its initial position until it first comes to rest.

4m

[4]



(c) Determine the time, correct to two decimal places, when the particle returns to its original position.

3m

[3]

(d) Calculate the acceleration of the particle when  $t = 2$ .

2m

[2]

**END OF TEST**





